ELEN E3401: Electromagnetics

Spring 2025

Prof. Keren Bergman
Department of Electrical Engineering
Columbia University

Lecture #1

COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

Course Outline

Lecture and Recitation:

Lectures: Mondays/Wednesdays 2:40pm – 3:55pm; Mudd 829

Recitations: Will be held as needed on Fridays 12:30pm – 2:00pm; Mudd 1300
Attendance in recitations is encouraged but not required

Contacts and Office Hours:

Prof. Keren Bergman, Schapiro/CEPSR 806, 212-853-1657 (bergman@ee.columbia.edu)
Office hours: Tuesdays 2:00pm – 3:00pm and by appointment.

TA: Brett George, CEPSR 821 (bcg2133@columbia.edu)
Office hours: Mondays and Wednesdays 1:00pm – 2:00pm and by appointment
OH location: CEPSR 8th Floor Lounge

Textbook:

Fundamentals of Applied Electromagnetics, 8th Edition, Ulaby and Ravaioli
Textbook Website: http://em8e.eecs.umich.edu/

Recommended References:

Elements of Engineering Electromagnetics, Nannapaneni Narayana Rao, 6th Edition
Engineering Electromagnetics, William H. Hayt, John A. Buck

Course Outline

Grading Structure:

1. Problems sets (9):	36%
2. Exams (2):	50%
3. Project:	14%

Grading Policy:

Problem sets must be submitted by 6pm to the TA on due date to be eligible for full credit. Late problem sets will be deducted 25% for day 1 and 50% for day 2. No problem sets will be accepted after 2 days. Exams are given during the regularly scheduled class time.

Course Schedule

Schedule

1.	Jan. 22 Wed.	Introduction, Electric and Magnetic fields (1-3 to 1-5)
2.	Jan. 27 Mon.	Traveling waves, complex domain and phasors (1-6, 1-7)
3.	Jan. 29 Wed.	Transmission lines, lumped element model (2-1, 2-2)
4.	Jan. 31 Fri.	Assign: prob. set #1
5.	Feb. 3 Mon.	Transmission line equations, wave propagation (2-3, 2-4)
6.	Feb. 5 Wed.	Lossless transmission line, microstrip (2-5, 2-6)
7.	Feb. 7 Fri.	Problem set #1 due; Assign: prob. Set #2
8.	Feb. 10 Mon.	Complex reflection coefficient, standing waves (2-6)
9.	Feb. 12 Wed.	Wave impedance of lossless line (2-7)
10.	Feb. 14 Fri.	Problem set #2 due; Assign: prob. Set #3
11.	Feb. 17 Mon.	Special cases, Impedance matching, power flow (2-8, 2-9)
12.	Feb. 19 Wed.	Transmission line transients; Vector fields (2-12; 3-1 to 3-7)
13.	Feb. 21 Fri.	Problem set #3 due; Assign: prob. Set #4
14.	Feb. 24 Mon.	Maxwell’s eq, Electrostatics, Gauss’s Law (4-1 to 4-4)
15.	Feb. 26 Wed.	Electric scalar potential; Conductors, dielectrics (4-5 to 4-7)
16.	Feb. 28 Fri.	Problem set #4 due; Assign: prob. Set #5

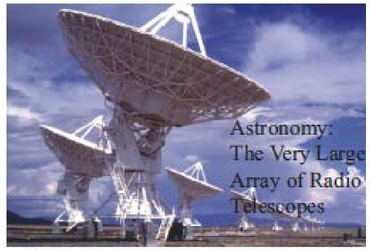
Course Schedule

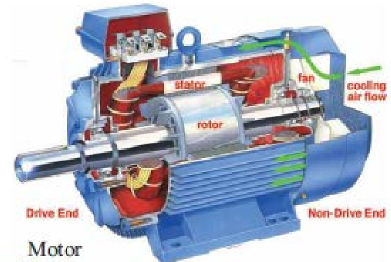
17. March 3 Mon.	Electric boundary conditions; capacitance, (4-8)
18. March 5 Wed.	Capacitance, Electrostatic potential energy (4-9, 4-10)
19. March 7 Fri.	Problem set #5 due
20. March 10 Mon.	Exam 1 review
21. March 12 Wed.	**Exam 1**
22. March 17 Mon.	SPRING BREAK
23. March 19 Wed.	SPRING BREAK
24. March 24 Mon.	Magnetic forces and Torques (5-1); Project assignment
25. March 26 Wed.	Magnetic fields, current distributions (5-2)
26. March 28 Fri.	Project team and topic due; Assign: prob. Set #6
27. March 31 Mon.	Magnetostatics, inductance (5-3, 5-7)
28. April 2 Wed.	Maxwell time-varying fields, Faraday's Law (6-1, 6-2)
29. April 4 Fri.	Problem set #6 due, Assign: prob. Set #7
30. April 7 Mon.	Transformer; moving conductor in magnetic field (6-3, 6-4) Project proposals due

Course Schedule

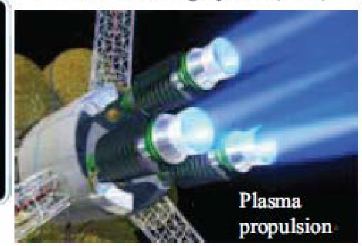
31. April 9 Wed.	Electromagnetic generator (6-5, 6-6)
32. April 11 Fri.	Problem set #7 due, Assign: prob. Set #8
33. April 14 Mon.	Time-harmonic fields, plane wave propagation (7-1, 7-2)
34. April 16 Wed.	Wave equation, polarization, lossy media (7-3, 7-4)
35. April 18 Fri.	Problem set #8 due, Assign: prob. Set #9
36. April 21 Mon.	Current flow in conductors, power density (7-5, 7-6)
37. April 23 Wed.	Waves at boundary, lossy media, TL analogy (8-1)
38. April 25 Fri.	Problem set #9 due
39. April 28 Mon.	Exam 2 review
40. April 30 Wed.	**Exam 2**
41. May 5 Mon.	Final projects presentations (part 1)
42. May 6 Tues	Final projects presentations (part 2)
43. May 9 Fri.	Project reports due

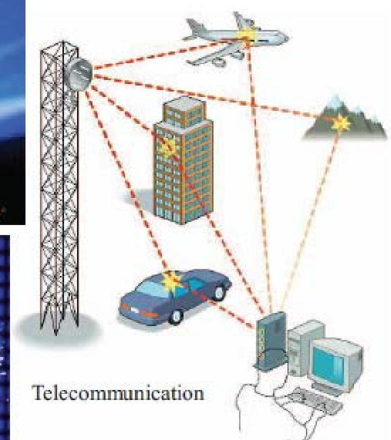
Numerous Systems and Applications



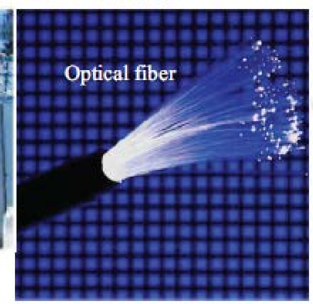


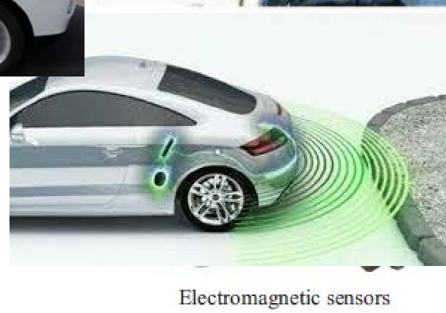
OLEDs

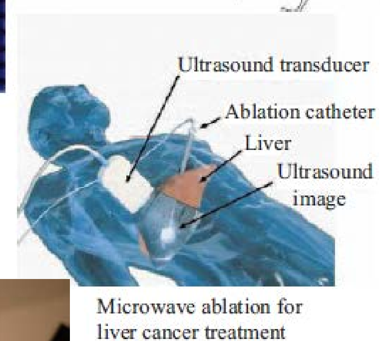


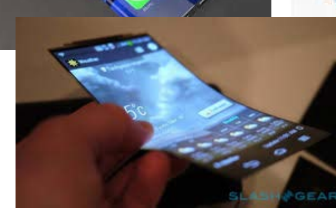


LiDAR

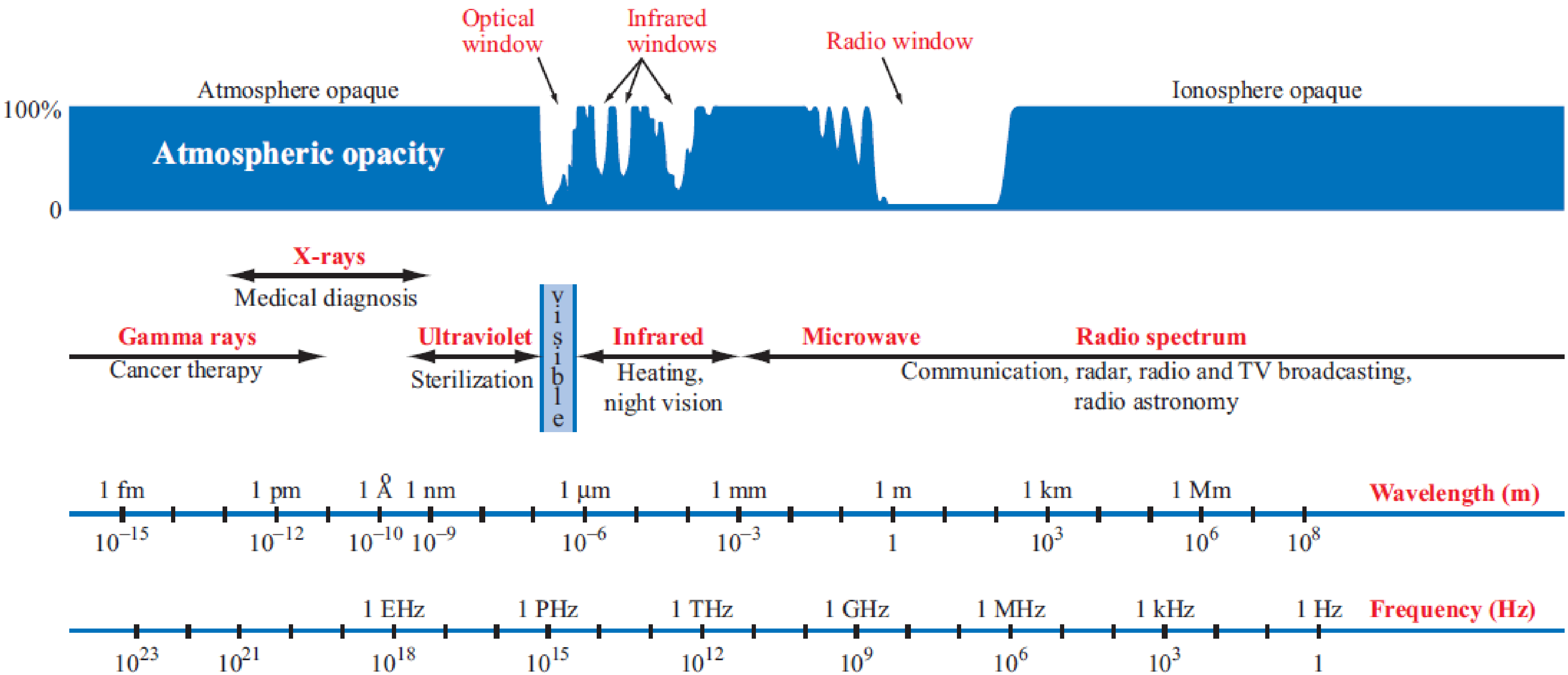








The EM Spectrum



$$\lambda = \frac{c}{f}$$

Dimensions and Units

Table 1-1: Fundamental SI units.

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol

Table 1-2: Multiple and submultiple prefixes.

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Charge: Electrical property of particles

Units: coulomb

One coulomb: amount of charge accumulated in one second by a current of one ampere.

1 coulomb represents the charge on ~ 6.241×10^{18} electrons

The coulomb is named for a French physicist, Charles-Augustin de Coulomb (1736-1806), who was the first to measure accurately the forces exerted between electric charges.

Charge of an electron

$e = 1.602 \times 10^{-19}$ C

Charge conservation

Cannot create or destroy charge, only transfer

Electrical Force

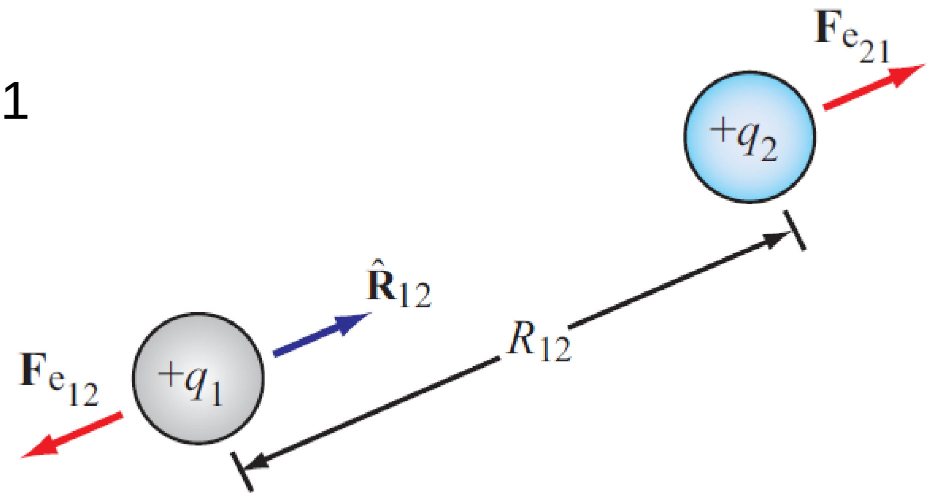
$q_e = -e$ (electron)

$q_p = e$ (proton)

Force exerted on charge 2 by charge 1

$$\vec{F}_{e_{21}} = \hat{R}_{12} \frac{q_1 q_2}{4\pi\varepsilon_0 R_{12}^2} \quad [N]$$

(free space)



R_{12} = distance between the 2 charges

$\hat{R}_{12}$ = unit vector pointing from q_1 to q_2

Force exerted on charge 2 by charge 1

$\vec{F}_{e_{21}} = -\vec{F}_{e_{12}}$ equal magnitude, opposite direction

Electric Field In Free Space

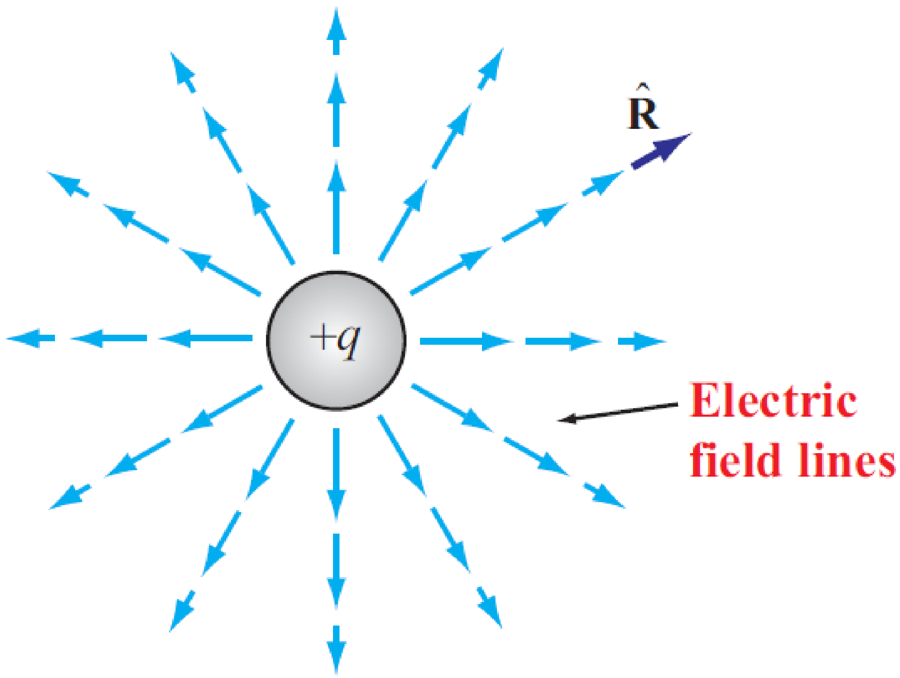
Force exerted on any point charge q’ present in an electric field (due to other charges) will experience a force:

$$\vec{F}_e = q'\vec{E}$$

$\vec{E}$ = Electric Field

$$\vec{E} = \hat{R}\frac{q}{4\pi\varepsilon_0 R^2} \quad \left[\frac{V}{m}\right] \text{ (in free space)}$$

Permittivity of free space



- Due to any charge in free space
- Charge force acts at a distance

$$\varepsilon_0 \approx 8.854 \times 10^{-12} \quad \left[\frac{F}{m}\right]$$

Electric Field

- Law of electric charge conservation

  Net electric charge cannot be created or destroyed

  Net charge remains unchanged

$$q = n_p e - n_e e = (n_p - n_e)e \quad [C]$$

- Principle of superposition

  Total vector electric field at a point in space due to a system of charges is equal to the vector sum of electric field at that point due to the individual charges

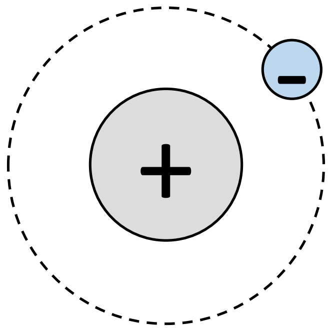
  Allows computing $\vec{E}$ due to complex charge distributions!

Dielectric Materials

The electric field due to a point charge in a material

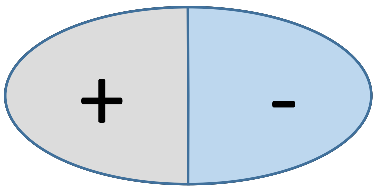
- Without charge

  Material is neutral, each atom represented as:



- With field from charge

  Atoms are polarized. Polarized atom leads to electric dipole:

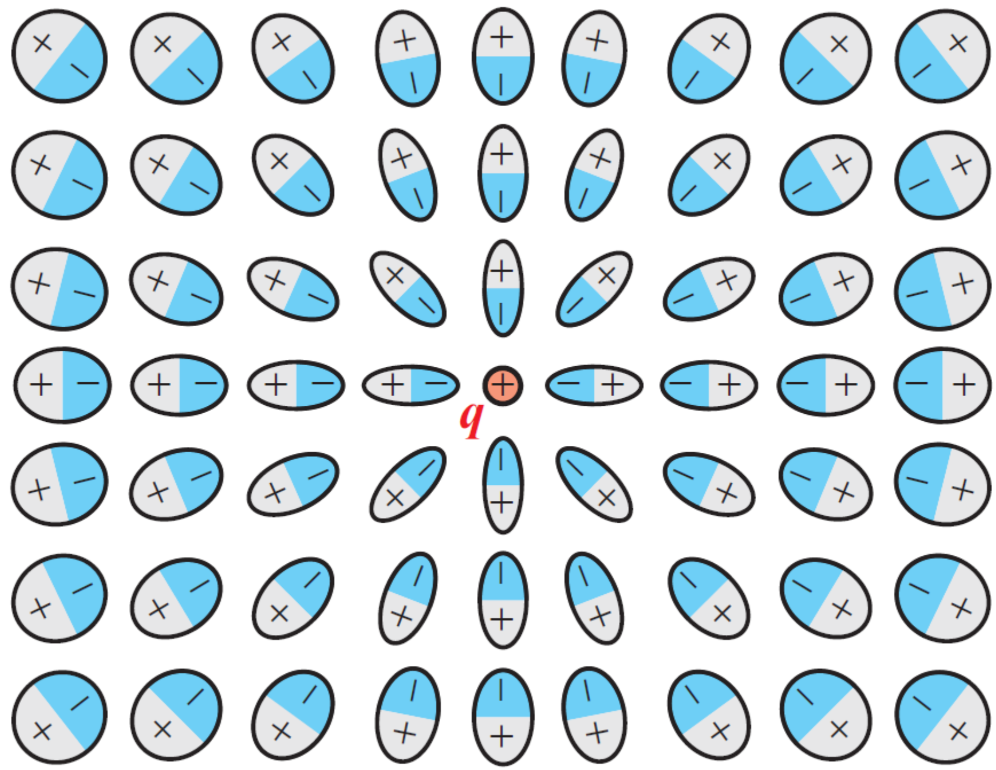


Electric Field Inside Dielectric Material

- Point charge *q* polarizes atoms in material.
- Electric field of dipoles will counteract field produced by point charge.

$$\vec{E} = \hat{R}\frac{q}{4\pi\boldsymbol{\varepsilon}R^2} \qquad \left[\frac{V}{m}\right]$$

ε =material permittivity



Relative permittivity and Electric Flux

$$\varepsilon = \varepsilon_r \varepsilon_0 \quad \left[\frac{F}{m}\right]$$

ε_r =relative/dielectric permittivity

Vacuum	$\varepsilon_r = 1$
Air	$\varepsilon_r = 1.0006$
Paper	$\varepsilon_r = 2 - 4$
Water (Distilled)	$\varepsilon_r = 81$

$\vec{D} =$ Electric flux density

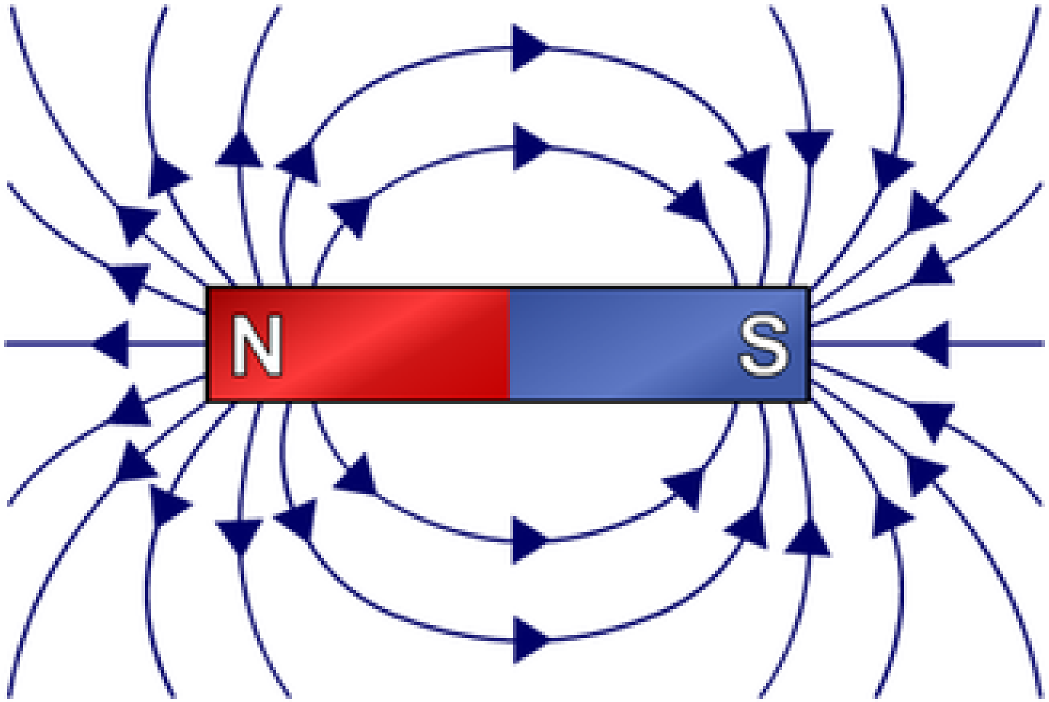
$$\vec{D} = \varepsilon \vec{E} \quad \left[\frac{C}{m^2}\right]$$

Magnetic Fields

Magnetic materials - Magnets

Magnetic poles always exist in pairs

Magnetic field lines



Magnetic field circulates from N to S

Magnetic force attracts/repels north-south poles

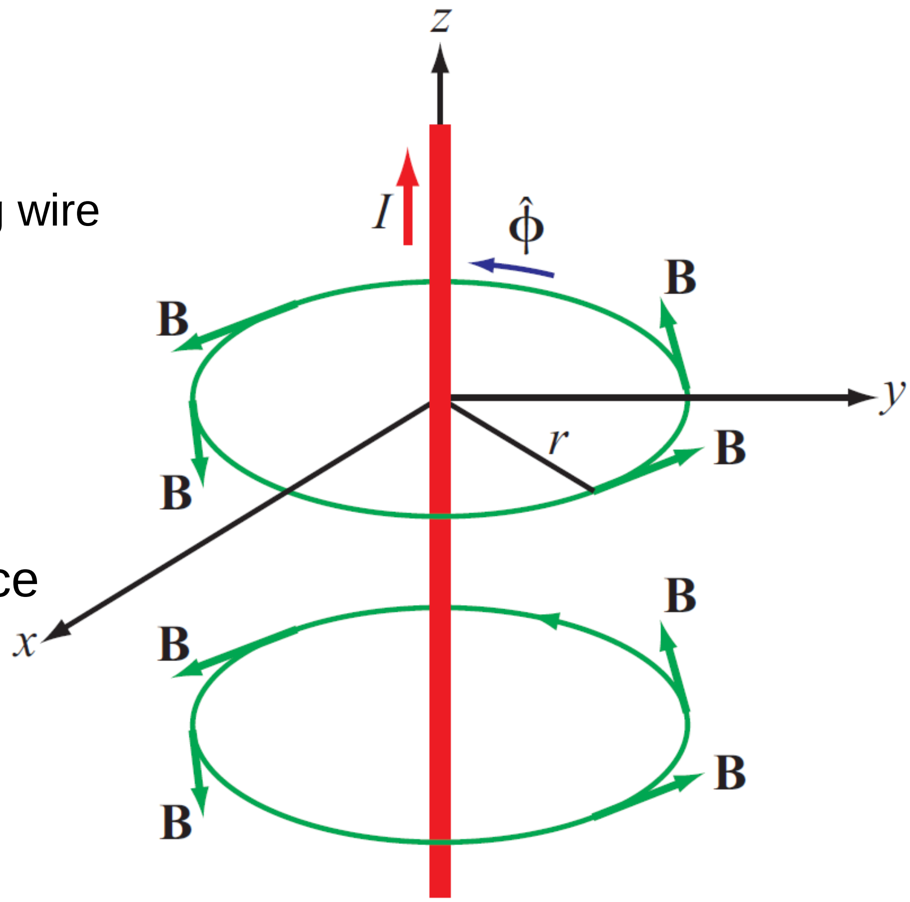
Magnetic Field

$\vec{B}$ = Magnetic Flux Density

$$\vec{B} = \hat{\phi}\frac{\mu_0 I}{2\pi r} \quad [T]$$

Magnetic field induced by current I in a long wire

- I = constant current
- $r =$ radial distance from wire
- $\hat{\phi} =$ azimuthal unit vector



μ_0 = Magnetic permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \quad \left[\frac{H}{m}\right]$$

Electric and magnetic fields connected through the speed of light c

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \quad \left[\frac{m}{s}\right]$$

Magnetic Force

Magnetic Force : $\vec{F}_m \rightarrow q'$ will experience magnetic force in magnetic flux **only** if charge is moving in non-parallel direction to $\vec{B}$

- $\vec{F}_m$ is $\perp$ (perpendicular) to both $\vec{B}$ and charge velocity $\vec{u}$

$\mu = \mu_r \mu_0 \left[\frac{H}{m}\right]$ - magnetic permeability

$\vec{B} = \mu \vec{H}$ $\quad\quad$ $\vec{H}$ = magnetic field intensity

Most materials not magnetic ($\mu_r \approx 1$)

Vacuum & Dielectrics	$\mu_r = 1$
Purified Iron	$\mu_r \approx 200{,}000$
Cobalt	$\mu_r = 250$
Iron (pure)	$\mu_r \approx 4{,}000 - 5{,}000$

Static vs Dynamic Fields

Static – does not vary with time (DC)

- $\vec{E}$ – governed by q
- $\vec{B}$ – governed by $I = \frac{\partial q}{\partial t}$

DC case:

$$I = \text{constant} \rightarrow \frac{\partial q}{\partial t} \neq 0 \rightarrow \frac{\partial I}{\partial t} = 0$$

$$\rightarrow \vec{B} \text{ not coupled to } \vec{E}$$

Static Fields

Electrostatics: Stationary Charges

$$\frac{\partial q}{\partial t} = 0$$

$$\vec{D} = \varepsilon \vec{E}$$

$\vec{E}$ $\left[\frac{V}{m}\right]$

$\vec{D}$ $\left[\frac{C}{m^2}\right]$

Magnetostatics: Steady Currents (DC)

$$\frac{\partial I}{\partial t} = 0$$

$$\vec{B} = \mu \vec{H}$$

$\vec{B}$ $[T]$

$\vec{H}$ $\left[\frac{A}{m}\right]$

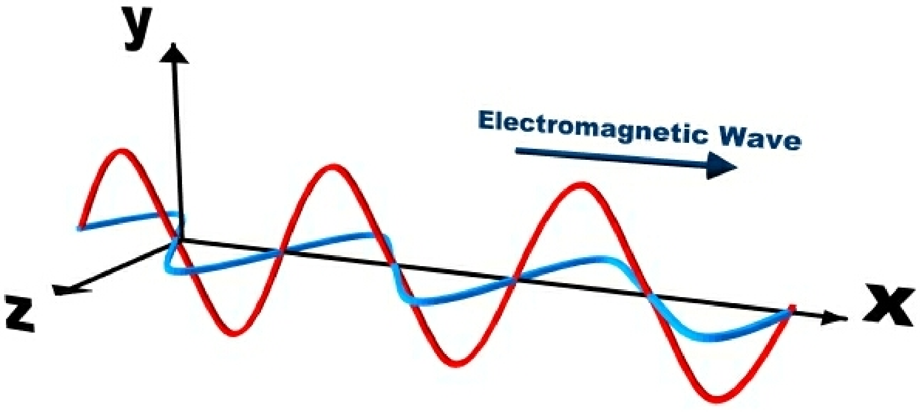
Dynamic Fields

Dynamics: Time-varying fields/current (AC)

$$\frac{\partial I}{\partial t} \neq 0$$

$$\left(\vec{E}, \vec{D}\right) \text{ coupled to} \left(\vec{B}, \vec{H}\right)$$

$$\text{Time varying } \vec{E} \leftrightarrow \text{generates } \leftrightarrow \text{Time varying } \vec{H}$$



Static vs. Dynamic Summary

Table 1-3: The three branches of electromagnetics.

Branch	Condition	Field Quantities (Units)
Electrostatics	Stationary charges $(\partial q/\partial t = 0)$	Electric field intensity $\mathbf{E}$ (V/m) Electric flux density $\mathbf{D}$ (C/m^2) $\mathbf{D} = \varepsilon\mathbf{E}$
Magnetostatics	Steady currents $(\partial I/\partial t = 0)$	Magnetic flux density $\mathbf{B}$ (T) Magnetic field intensity $\mathbf{H}$ (A/m) $\mathbf{B} = \mu\mathbf{H}$
Dynamics (Time-varying fields)	Time-varying currents $(\partial I/\partial t \neq 0)$	$\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$, and $\mathbf{H}$ ($\mathbf{E}$, $\mathbf{D}$) coupled to ($\mathbf{B}$, $\mathbf{H}$)

Under static conditions, electric and magnetic fields are independent, but under dynamic conditions, they become coupled.

Material Properties

Parameter	Symbol	Value	Units
Electrical Permittivity (Free Space)	ε_0	$\approx 8.854 \times 10^{-12}$ $\approx \frac{1}{36\pi} \times 10^{-9}$	$\frac{F}{m}$
Magnetic Permeability (Free Space)	μ_0	$= 4\pi \times 10^{-7}$	$\frac{H}{m}$
Conductivity (Free Space or Perfect Dielectric)	σ	$= 0$	$\frac{S}{m}$
Conductivity (Perfect Conductor)	σ	$= \infty$	$\frac{S}{m}$

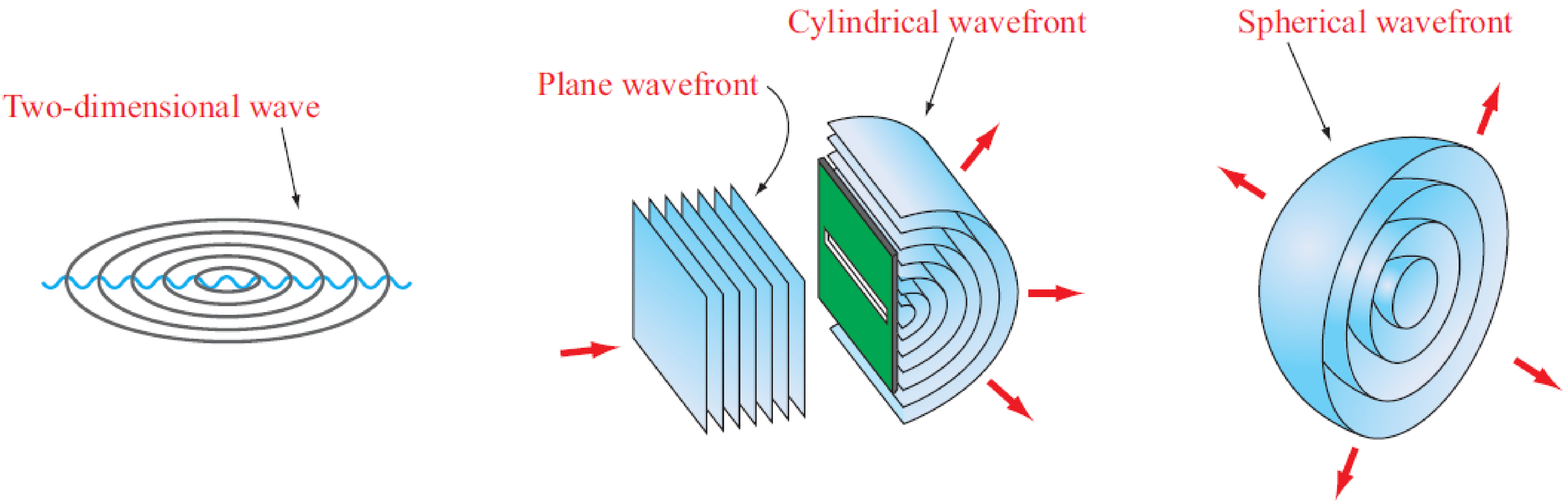
Constitutive parameters

Traveling Waves

- Waves carry energy
- Waves have velocity
- Many waves are linear: they do not affect the passage of other waves; they can pass right through them

- Transient waves: caused by sudden disturbance
- Continuous periodic waves: repetitive source

Traveling Waves



(a) Circular waves — 2D

(b) Plane and cylindrical waves — 3D

(c) Spherical wave — 3D

Vacuum velocity of EM Wave

$$c_0 = 2.99792458 \times 10^8 \approx 3 \times 10^8 \quad \left[\frac{m}{s}\right]$$

Wave propagation

Consider a 2D wave, sinusoidal, in a lossless medium:

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)$$

A = Amplitude

T = Time Period [s] $\left(f = \frac{1}{T} \quad \left[\frac{1}{s} \text{ or Hz}\right]\right)$

λ = Spatial Wavelength [m]

ϕ_0 = Reference Phase [rad]

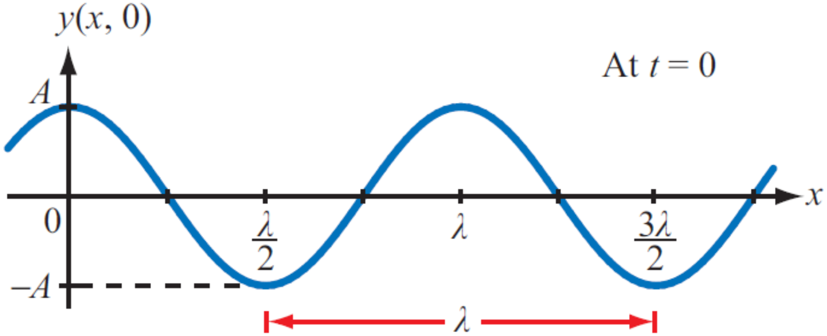
*A medium is said to be **lossless** if it does not attenuate the amplitude of the wave traveling within it or on its surface.*

2D Sinusoidal wave in lossless medium

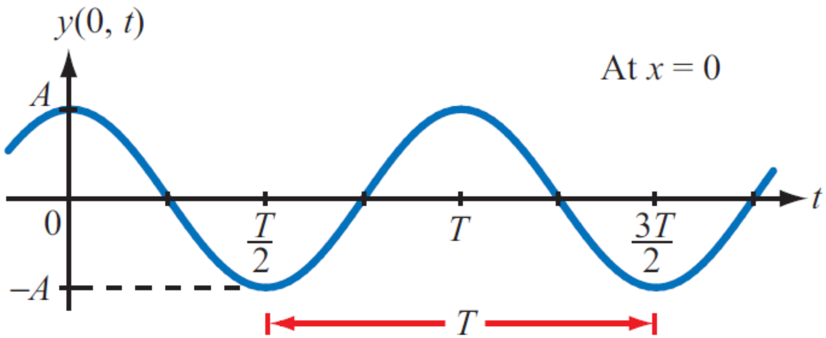
y = height of water surface
x = distance

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)$$

Let ϕ_0=0



(a) $y(x, t)$ versus x at $t = 0$



(b) $y(x, t)$ versus t at $x = 0$

Wave propagation – phase velocity

Consider 2D wave:

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right) = A\cos\big(\phi(x,t)\big)$$

Wave peaks at: $\phi(x,t) = n \times 2\pi$ (n is integer)

$$\phi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 = 2n\pi$$

$$\phi(x,t) = 2n\pi$$

$$\rightarrow y(x,t) = \text{constant} = y_0$$

In general, the constant phase is: $\cos^{-1}\left(\frac{y_0}{A}\right) = \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$

Wave propagation – phase velocity

What is the velocity of the wave front (constant phase):

$$\frac{\partial}{\partial t}\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) = \frac{2\pi}{T} - \frac{2\pi}{\lambda}\frac{\partial x}{\partial t} = 0$$

$$\frac{\partial x}{\partial t} = \frac{\lambda}{T}$$

We define, phase velocity, u_p:

$$u_p = \frac{\partial x}{\partial t} = \frac{\lambda}{T} = f\lambda = \frac{\omega}{\beta} \quad \left[\frac{m}{s}\right]$$

Angular Frequency, ω

$$\omega = 2\pi f \quad \left[\frac{rad}{s}\right]$$

Phase Constant, β:

$$\beta = \frac{2\pi}{\lambda} \quad \left[\frac{rad}{m}\right]$$

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$y(x,t) = A\cos(\omega t + \beta x + \phi_0)$$

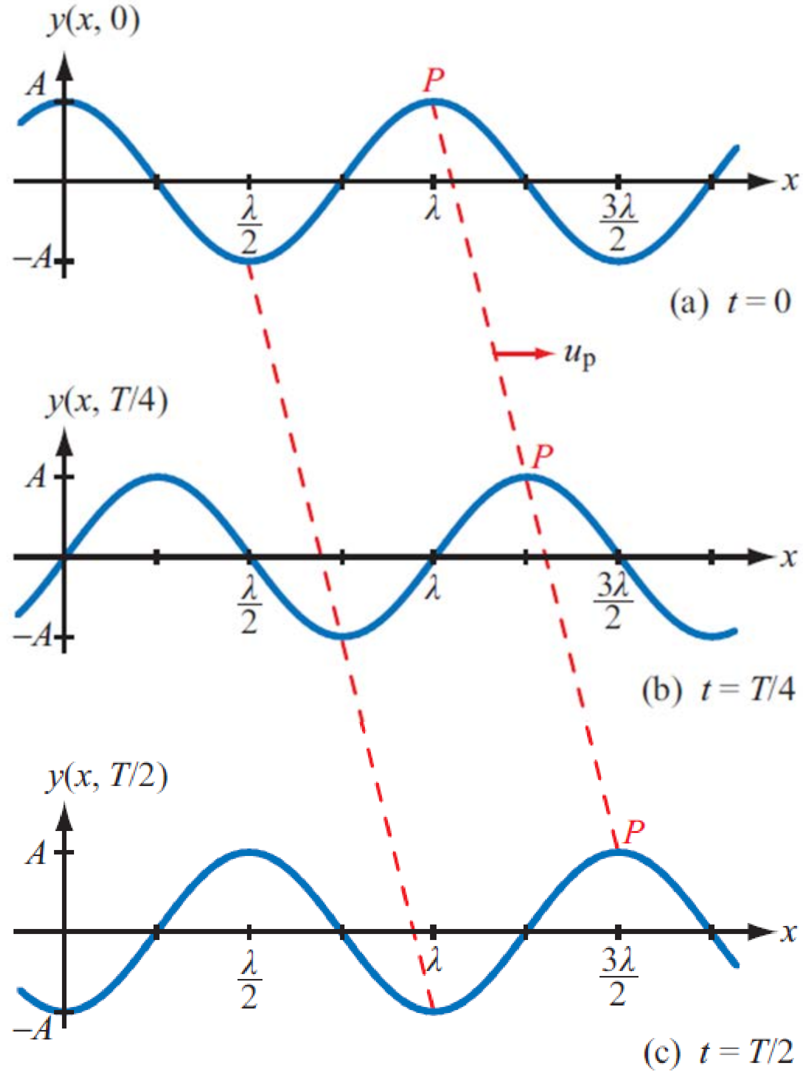
Wave traveling in $+\hat{x}$ direction:

$-\hat{x}$ direction:

Phase velocity

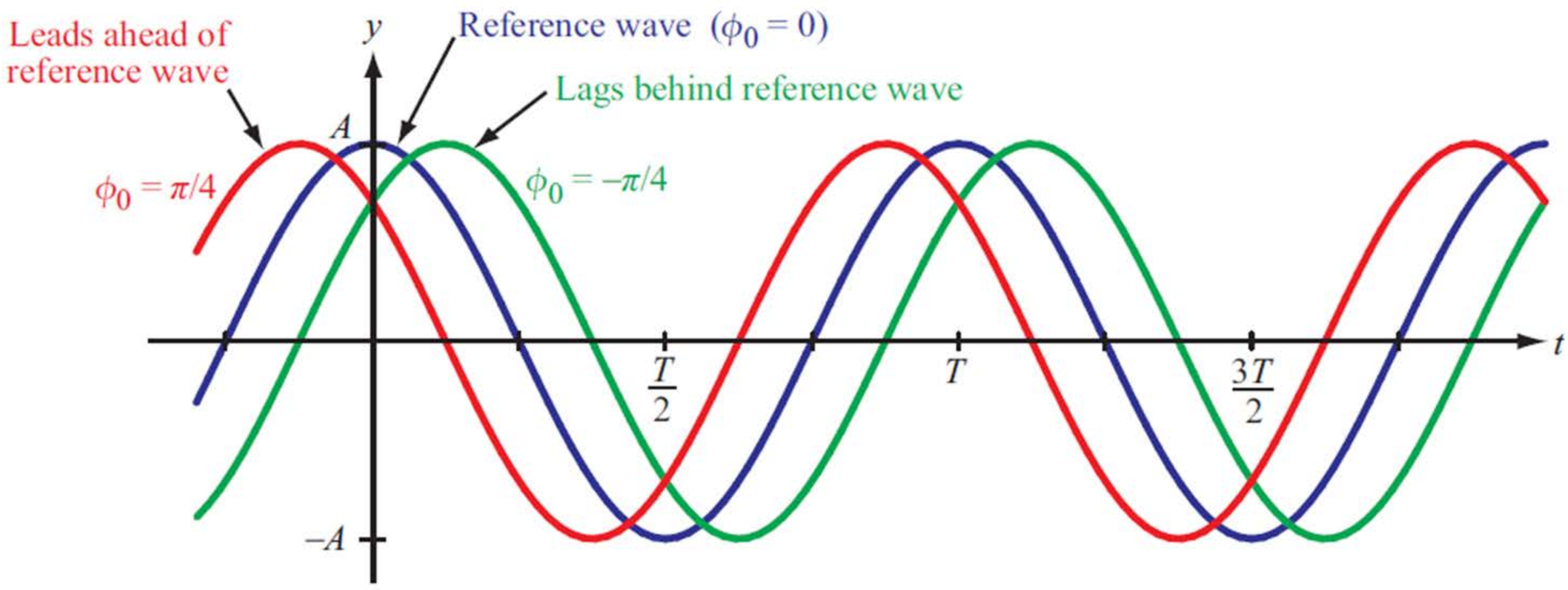
$$y(x,t) = A\cos\big(\phi(x,t)\big)$$

$$\phi(x,t) = \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)$$



Phase Lead and Lag

$y(0,t) = A\cos(2\pi t/T + \phi_0)$ 3 values for ϕ_0



$y(z,t) = A\cos(\omega t - \beta z + \phi_0)$

Positive reference phase, ϕ_0, corresponds to a phase lead, while negative ϕ_0 corresponds to a phase lag

Lossy Medium

Attenuation Factor

$$y(z,t) = A\overbrace{\boldsymbol{e^{-\alpha z}}}\cos(\omega t - \beta z + \phi_0) \qquad \alpha = \text{ attenuation constant } \left[\frac{Np}{m}\right]$$